DEVELOPMENT OF MATHEMATICAL MODEL OF CUT HAULM MOVEMENT ALONG WORKING TOOL OF LOADING MECHANISM

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Abstract. A new design of the haulm removing machine has been developed, in which the loading mechanism is designed in the form of a spinner, to which the entire volume of the cut sugar beet haulm is fed, and the upper discharge chute is at the level of the vehicle body. In order to substantiate the kinematic and design parameters of this loading device of the cut haulm, there was developed a mathematical model of the movement of a particle of the sugar beet haulm along the blade of the spinner. At the same time the blades of this spinner of the loading mechanism ensured sliding of the cut particles of the haulm along the inner surface of the cylindrical part of the casing and along its rectilinear part. In further solution of the obtained theoretical dependencies on the PC according to a compiled program, graphical dependencies were obtained, which provided a possibility to substantiate rational parameters of the working tools of the loading mechanism of a haulm removing machine that performs continuous cutting of the sugar beet haulm without a tracer.

Keywords: sugar beet, haulm, cutting, loading, simulation.

Introduction

Sugar beet is the most important agricultural crop in many countries of the world [1-3]. Besides the sugar-bearing mass of the roots, the sugar beet yield includes its haulm, which is valuable forage or a raw material for processing into biogas. By its composition the sugar beet haulm can be compared with high-quality green fodder [4]. One centner of freshly harvested haulm contains 20 fodder units, 2.2 kg of digestible protein, 2.5 kg of calcium, 0.5 kg of phosphorus [5]. Therefore, the technological operation of efficient and high-quality removal of the haulm from the heads of the sugar beet roots without extracting them from the ground in the process of harvesting is an actual practical and scientific problem.

The most time-consuming operation is harvesting of the beets. In the process of harvesting it is also necessary to remove the haulm from the root portion and transport it into the vehicles. In addition, the haulm must not be polluted by soil, and its cut from the root crop should ensure minimal (according to the local agrodemands) loss of the sugar-bearing mass.

The general theory of the movement of a material particle along the working surfaces of the agricultural machines was studied in the fundamental works by P. Vasilenko P., Bulgakov V. et al. [6-9]. In many other works these concepts are developed in the aspect of clarifying individual components [10]. The movement of the haulm along the working surface of the conveyor has a number of peculiarities, and it also requires specification of the well-known theoretical concepts. In connection with the development of new types of the working tools of the machines used in agriculture under contemporary conditions, these well-known concepts require certain alterations that are associated not only with the design features of these working tools, but also with bringing of the cumbersome equations to a closed form which will be convenient for the use in further simulation.

The purpose of the work is to substantiate the rational design and kinematic parameters of the loading mechanism of sugar beet haulm to ensure efficient loading into the vehicle body.

Materials and methods

When conducting the theoretical research, methods of higher mathematics, theoretical mechanics, as well as methods of compiling computer programs and analysing the results of calculations on the PC were used.

A haulm removing machine has been developed with a loading mechanism of a variable design, which is a blade spinner with a possibility to use blades of various geometrical shapes [11]. Fig. 1 shows the design and technological scheme of the haulm removing machine with an improved loading mechanism; in Fig. 2 there is given a view of operation of this machine under the field conditions.

The design of the haulm removing machine carries out continuous cutting without a tracer by means of a rotary haulm cutting apparatus 3. Then the sugar beet haulm, cut to the entire working

width of the machine, is collected, transported to the side part of the haulm removing machine and fed to the loading mechanism, which consists of a blade spinner 4. Here the particles of the haulm fall onto the blades of the spinner 4 and, through the loading chute 5 are loaded into the body of the vehicle, running alongside with the haulm removing machine. The spinner blades 4 mechanically throw the particles of the haulm into the loading chute 5; in addition, the blades 4 create air pressure that also contributes to efficient transportation and loading of the removed haulm into the vehicle.
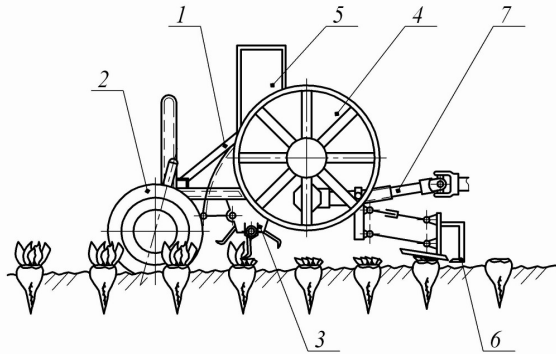

Fig. 1. **Design and technological scheme of haulm removing machine with improved loading mechanism:** 1 – frame; 2 – tracer wheel; 3 – rotary haulm cutting apparatus; 4 – blade spinner of the haulm; 5 – loading chute; 6 – pre-cutter of the root heads; 7 – drive of the working tools

Fig. 2. **Operation of haulm removing machine with improved loading mechanism under field conditions**

In order to substantiate rational parameters of the blade spinners of the loading mechanism, it is necessary to construct a computational mathematical model of the haulm loading process. For this purpose we construct an equivalent scheme at the beginning. Let us consider the cross section of the mechanism for loading the haulm, which is a cylindrical casing with the radius R, in which a blade spinner is installed on the drive shaft with the radius r_o. The spinner has four blades fixed on the drive shaft, located tangentially to the flange, i.e. at some angle to the radial direction. During the rotation of the shaft of the spinner each of the blades successively approaches the zone of the loading window, capturing a certain amount of the haulm coming in at this time and involving it by the blade in joint movement. After that the particles of the haulm, captured by each blade, move forward along the surface of the blade itself to its end and participate at the same time in the rotational movement together with the blade. This movement of the haulm particles ends after the blade reaches the discharge window, when it is thrown upwards into the loading chute. Thus, from the moment when the haulm particles are captured to their throwing into the loading chute, their movement takes place actually in a closed space limited by two adjacent blades and the lower guiding casing of the blade spinner.

For simplicity, we will show only one of the blades in an equivalent scheme, assuming that the others are in the same position [7;12]. Let us designate the centre of rotation of the considered system by point O; the length of the blade by AB. The direction of rotation of the blade is shown by an arrow.

Consider analytically the movement of a particle of the haulm M along the blade of the spinner. At the initial moment of time we will show the blade in a position when its outer end is at the lower point of the trajectory, i.e. at point B. Let the blade has turned for some time t in the direction of rotation, but its lower point B has moved along the arc of a circle and has taken position B' (Fig. 3). Consequently, the haulm particle M, being some time t on the surface of the blade that had captured it in a random position, will turn at an angle equal to φ, which $\varphi = \omega t$ (where ω – angular speed).

Let us show the angular parameters of this mechanical system, for which we introduce the following notation. Let ψ be the angle between the plane of the blade and the radius drawn through the axis of rotation and point M, which corresponds to the position of a haulm particle on the blade at a random moment of time t, in its any position. In addition, ψ_o is the initial value of the angle ψ, i.e. , the

value of the angle ψ in the position, when point M coincides with point A (or A' when the blade is in a random position), in other words, in a position when the haulm particle M is at point A (or at point A' in a random position of the blade). In the initial position of the blade (indicated as AB in Fig. 3) ψ_o is , in fact, an angle between the plane of the blade and the horizontal radius. Obviously, when the blade rotates, this angle remains constant, that is $\psi_o = $ const. In this case, ψ_1 – the final value of angle ψ, that is, the value of the angle ψ in the position, when point M coincides with point B (or point B' in a random position of the blade). In the initial position of the blade angle ψ_1 is the angle between the plane of the blade and the vertical radius. Obviously, when the blade rotates, this angle also remains constant, i.e. $\psi_1 = $ const. Consequently, when the particle of the haulm M is moving from point A (A') to point B (B') regardless of the rotation of the blade, the angle ψ varies within the range from ψ_o to ψ_1 , i.e., $\psi_1 \leq \psi \leq \psi_o$. Besides, β is an angle between the plane of the blade and the vertical in any moment of time t . As it is evident from the equivalent scheme of Fig. 3, $\beta = \omega t - \psi_1$.
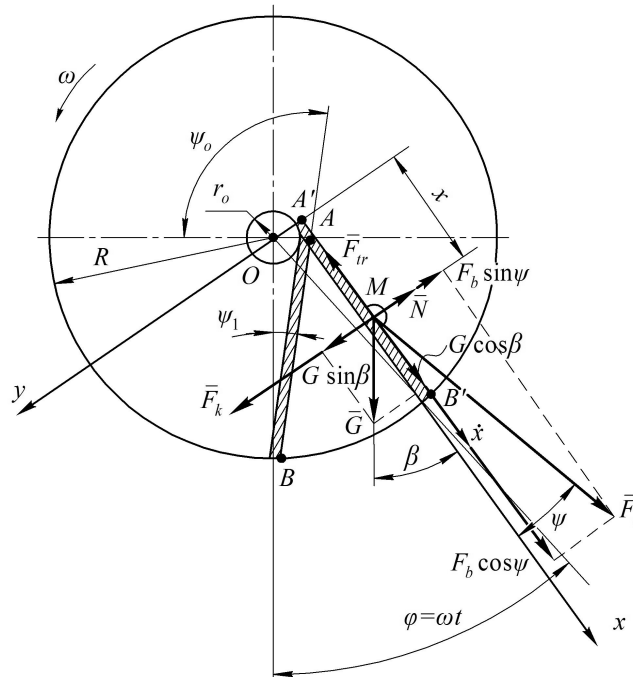

Fig. 3. **Equivalent scheme of movement of haulm particle along blade of loading mechanism**

Let us introduce the plane Cartesian coordinate system xAy, rigidly connected with the blade. In this case, we will direct axis x along the blade from the place where it is fixed on the drive shaft (point A), and axis y perpendicular to the plane of the blade, and we will show it passing through the centre of rotation O (Fig. 3).

We will determine the necessary geometrical correlations between the parameters of the particular loading mechanism, presented in the equivalent scheme of Fig. 3. So:

$$x = r \cdot \cos\psi - r_o \cdot \cos\psi_o, \tag{1}$$

and

$$r \cdot \sin\psi = r_o \cdot \sin\psi_o = \text{const}, \tag{2}$$

where r – the radius of the position of point M relative to point O.

Let us consider the relative movement of the sugar beet haulm particle M along the blade in the moving coordinate system xAy.

To compile a differential equation of the movement of the haulm particle M along the surface of the blade it is necessary to take into account all the forces that act upon the particle during its movement. Let us designate successively the forces applied to point M.

We will assume that the haulm particle M falls onto the blade at an initial speed $V_o = 0$. Obviously, when the disk rotates around the horizontal axis and the particle moves along the blade from the centre of the disk to its edge, the forces acting upon the particle M (Fig. 3) are:

1) force of gravity $\bar{G}$ of the haulm particle M, equal to:

$$G = mg ,\qquad(3)$$

where m – mass of the particle;
        g – acceleration of gravity;

2) centrifugal force $\bar{F}_b$, which will be equal to:

$$F_b = m \cdot a^n = m \cdot r \cdot \omega^2 ,\qquad(4)$$

where $a^n = r \cdot \omega^2$ – normal acceleration of the sugar beet haulm particle M;

3) Coriolis inertial force $\bar{F}_k$, which has the following value:

$$F_k = m \cdot a^k = 2m \cdot \omega \cdot \dot{x} ,\qquad(5)$$

where $\dot{x}$ – relative speed of particle M along the blade;
        $a^k = 2\,\omega \cdot \dot{x}$ – Coriolis acceleration;

4) friction force $\bar{F}_{tr}$, the value of which will be equal to:

$$F_{tr} = f \cdot N ,\qquad(6)$$

where N – normal reaction of the surface of the blade,
        f – coefficient of friction.

According to the obtained scheme of forces, it is obvious that in this case a flat system of forces acts upon the sugar beet haulm particle M.

Using the scheme of forces acting upon the particle M, and, since the relative movement of this particle occurs only along the surface of the blade, i.e. only straightforward along the axis x, it is possible to compile a differential equation of its movement, applying the basic law of dynamics, in the following form:

$$m\ddot{x} = \sum_{k=1}^{n} F_{kx} ,\qquad(7)$$

where $\ddot{x}$ – relative acceleration of the particle M as it moves along the axis Ax;

$\sum_{k=1}^{n} F_{kx}$ – sum of the projections of all forces acting upon the particle at a random point in

time t.

Next, we project all the forces that act upon the haulm particle M onto the axis Ax. In this case the sum of the projections of all forces onto axis Ax will be equal to:

$$\sum_{k=1}^{n} F_{kx} = m \cdot r \cdot \omega^2 \cdot \cos\psi + mg \cdot \cos\beta - f \cdot N .\qquad(8)$$

In order to find the friction force F_{tr}, it is necessary to determine the normal reaction of the haulm particle N. To find it, we compose the algebraic sum of the projections of all forces onto the axis Ay, which, from the equilibrium condition, that is, from the condition of uninterrupted movement along the surface of the blade, should be equal to zero. We have:

$$\sum_{k=1}^{n} F_{ky} = 2m \cdot \omega \cdot \dot{x} + mg \cdot \sin\beta - m \cdot r \cdot \omega^2 \cdot \sin\psi - N = 0 ,\qquad(9)$$

from where we find the value of the sought-for force N:

$$N = 2m \cdot \omega \cdot \dot{x} + mg \cdot \sin\beta - m \cdot r \cdot \omega^2 \cdot \sin\psi .\qquad(10)$$

Then the friction force F_{tr}, which occurs when the particle M is moving along the blade, will be equal to:

$$F_{tr} = f \cdot N = f\left(2m \cdot \omega \cdot \dot{x} + mg \cdot \sin\beta - m \cdot r \cdot \omega^2 \cdot \sin\psi\right). \tag{11}$$

Results and discussion

By substituting all the determined values of the forces into expression (7), we obtain a differential equation relative to the movement of the haulm particle M along the blade in the following form:

$$m\ddot{x} = m \cdot r \cdot \omega^2 \cdot \cos\psi + m \cdot g \cdot \cos\beta -$$
$$-f\left(2m \cdot \omega \cdot \dot{x} + m \cdot g \cdot \sin\beta - m \cdot r \cdot \omega^2 \cdot \sin\psi\right). \tag{12}$$

Next, it is necessary to convert the differential equation (12). At first we will express the value of the radius r through the coordinate x. From correlation (1) we obtain:

$$r \cdot \cos\psi = x + r_o \cdot \cos\psi_o. \tag{13}$$

After substitution of expressions (2) and (13) into equation (12), and also considering correlation $\beta = \omega t - \psi_1$, we will have:

$$m\ddot{x} = m \cdot \omega^2\left(x + r_o \cos\psi_o\right) + mg \cdot \cos\left(\omega t - \psi_1\right) -$$
$$-2f \cdot m \cdot \omega \cdot \dot{x} - f \cdot m \cdot g \cdot \sin\left(\omega t - \psi_1\right) + f \cdot m \cdot \omega^2 \cdot r_o \cdot \sin\psi_o, \tag{14}$$

or

$$\ddot{x} + 2f \cdot \omega \cdot \dot{x} - \omega^2 \cdot x = r_o \cdot \omega^2 \cdot \cos\psi_o + g \cdot \cos\left(\omega t - \psi_1\right) -$$
$$-f \cdot g \cdot \sin\left(\omega t - \psi_1\right) + f \cdot r_o \cdot \sin\psi_o \cdot \omega^2. \tag{15}$$

Next, we transform equation (15) to a form convenient for integration. We will have:

$$\ddot{x} + 2f \cdot \omega \cdot \dot{x} - \omega^2 \cdot x = \left[g \cdot \cos\left(\omega t - \psi_1\right) - f \cdot g \cdot \sin\left(\omega t - \psi_1\right)\right] +$$
$$+r_o \cdot \omega^2\left(\cos\psi_o + f \cdot \sin\psi_o\right). \tag{16}$$

Now we need to open the brackets in the right side of equation (16). We have:

$$\ddot{x} + 2f \cdot \omega \cdot \dot{x} - \omega^2 \cdot x = g \cdot \cos\omega t \cdot \cos\psi_1 + g \cdot \sin\omega t \cdot \sin\psi_1 -$$
$$-f \cdot g \cdot \sin\omega t \cdot \cos\psi_1 + f \cdot g \cdot \cos\omega t \cdot \sin\psi_1 + r_o \cdot \omega^2\left(\cos\psi_o + f \sin\psi_o\right). \tag{17}$$

Let us rearrange the expression on the right side of equation (17) as follows:

$$\ddot{x} + 2f \cdot \omega \cdot \dot{x} - \omega^2 \cdot x = g \cdot \cos\omega t \cdot \left(\cos\psi_1 + f \cdot \sin\psi_1\right) +$$
$$+g \cdot \sin\omega t\left(\sin\psi_1 - f \cdot \cos\psi_1\right) + r_o \cdot \omega^2\left(\cos\psi_o + f \cdot \sin\psi_o\right). \tag{18}$$

We introduce the following notation:

$$L = g\left(\sin\psi_1 - f\cos\psi_1\right), \quad K = g\left(\cos\psi_1 + f\sin\psi_1\right), \quad D = \cos\psi_o + f\sin\psi_o \tag{19}$$

Then the differential equation (18) with coefficients (19) can be reduced to this form:

$$\ddot{x} + 2f \cdot \omega \cdot \dot{x} - \omega^2 \cdot x = L \cdot \sin\omega t + K \cdot \cos\omega t + r_o \cdot \omega^2 \cdot D. \tag{20}$$

Equation (20) is a second-order linear differential equation with the right side.

The general solution of the differential equation (20), which satisfies the initial conditions, will have the following form:

$$x = \frac{\lambda_2(T+Q) - S \cdot \omega}{\lambda_1 - \lambda_2} \cdot e^{\lambda_1 t} - \frac{\lambda_1(T+Q) - S \cdot \omega}{\lambda_1 - \lambda_2} e^{\lambda_2 t} +$$
$$+ S \cdot \sin \omega t + T \cdot \cos \omega t + Q. \tag{21}$$

Thus, the obtained expression (21) is the law of the movement of the haulm particle M along the blade AB at a random point of time t.

If the length of the blade $AB = l$, then from expression (21) it is possible to determine the time t_1 when the haulm particle M leaves the surface of the blade, provided that $x = l$.

Such a calculation can be performed on the PC by selecting the necessary step for the parameter t, so that for some $t = t_1$ equality $x = l$ holds true.

If we differentiate expression (21) by time t, we will obtain:

$$\dot{x} = \frac{\lambda_1\lambda_2(T+Q) - S \cdot \lambda_1 \cdot \omega}{\lambda_1 - \lambda_2} \cdot e^{\lambda_1 t} - \frac{\lambda_1\lambda_2(T+Q) - S \cdot \lambda_2 \cdot \omega}{\lambda_1 - \lambda_2} e^{\lambda_2 t} +$$
$$+ S \cdot \omega \cdot \cos \omega t - T \cdot \omega \cdot \sin \omega t. \tag{22}$$

Expression (22) is the law of the change in the relative speed of the movement of the haulm particle M along the blade AB at a random point in time t.

Substituting into (22) time $t = t_1$, one can determine the relative speed $\dot{x}_1 = \dot{x}(t_1)$, at which the haulm particle M leaves the surface of the blade AB.

After the relative speed $\dot{x}_1$ is determined at which the haulm particle M leaves the surface of the blade AB, it is necessary to determine its absolute speed of flight at the moment of leaving the disk of the spinner. Since the transfer speed of the haulm particle M is directed tangentially to the disk at the point of its leaving this disk and by the value is equal to ($\omega \cdot R$), but the angle between the vector of the relative and the transfer speeds is equal to ($90^\circ - \psi_1$), then by the cosine theorem we find the value of the absolute speed V_a of the haulm particle M when leaving the disk, which will be equal to:

$$V_a = \sqrt{\dot{x}_1^2 + \omega^2 \cdot R^2 - 2\dot{x}_1 \cdot \omega \cdot R \cdot \sin \psi_1}. \tag{23}$$

The resulting differential equation of the movement (20) holds for a rectilinear blade, located at an angle ψ_1 to the radial position. If the blade is located along the radius, i.e. $\psi_1 = 0$, then $\psi = 0$, $\psi_o = \pi/2$, the task is significantly simplified.

In this case the differential equation (20) assumes the following form:

$$\ddot{x} + 2f \cdot \omega \cdot \dot{x} - \omega^2 \cdot x = -g \cdot f \cdot \sin \omega t + g \cdot \cos \omega t + f \cdot r_o \cdot \omega^2. \tag{24}$$

In this way a mathematical model was constructed for the movement of a cut-off sugar beet haulm particle along a cleaning blade, i.e. along the working tool of the loading mechanism of the haulm removing machine.

For numerical simulation of this mathematical model on the PC we compiled a program of numerical calculations on the PC in the Matlab system. On the basis of the results of numerical calculations on the PC, there were constructed graphs of dependencies of the absolute speed V_a of the haulm particle M, leaving the end of the blade, upon the length l of the blade itself (Fig. 4), as well as similar graphs of dependencies of the absolute speed V_a of the haulm particle M, leaving the end of the blade, upon the angular velocity of rotation ω of its drive shaft (Fig. 5).

As evident from the presented graphs, the values of the absolute speed V_a of the haulm particle M, leaving the end of the blade, depending on the on the length of the blade l, have dependencies close to the linear ones. In addition, the maximum value of the linear speed V_a of the haulm particle M, leaving the end of the blade, is equal to 60 m·s^{-1}; taking into account the design limitations upon the diameter of the blade spinner, the drive shaft of which is coaxial with the feeding screw conveyor, the length of the cleaning blade l should not be greater than 0.45 m. Besides, a smaller value of the friction coefficient f will also contribute to the increase in the absolute speed of the haulm particle M when leaving the end of the blade (Fig. 4).
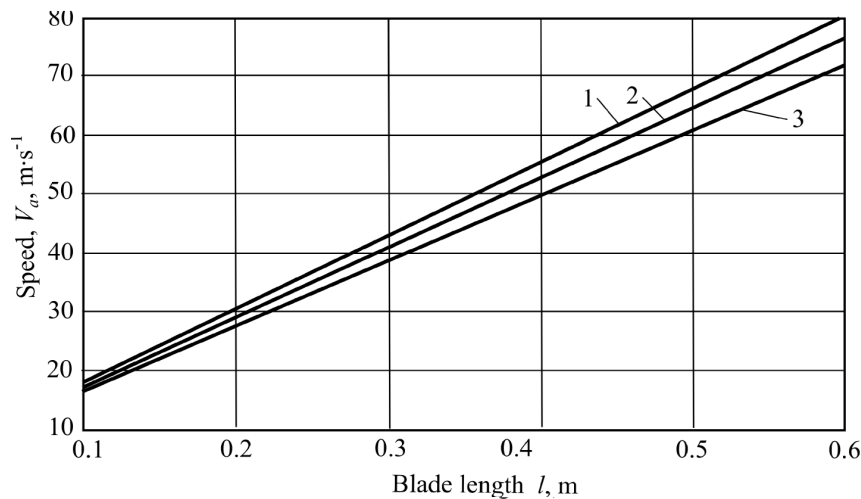
Fig. 4. Dependence of absolute speed V_a of haulm particle M, when leaving end of blade, upon length l of blade: $1 - f = 0.3$; $2 - f = 0.5$; $3 - f = 0.8$
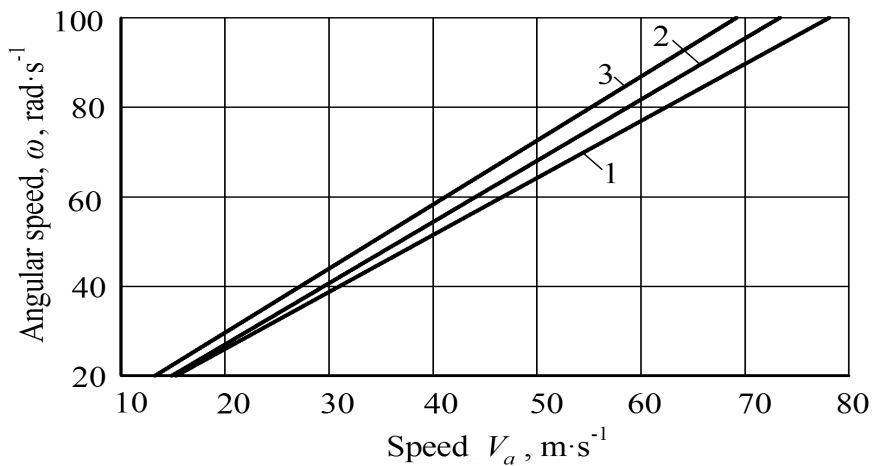

Fig. 5. Dependence of absolute speed V_a of haulm particle M, when leaving end of blade, upon angular speed ω of rotation of its drive shaft: $1 - f = 0.3$; $2 - f = 0.5$; $3 - f = 0.8$

The graphs, presented in Fig. 5 and showing the dependence of the maximum speed V_a of the haulm particle M, when leaving the end of the blade, upon the angular speed ω of rotation of the drive shaft of the blade spinner, also vary according to a law, close to the linear one. However, considering the limitations connected with the properties of the haulm, leading to its repeated disintegration during transportation and unloading, it is the angular speed ω, which is equal to 85 rad·s^{-1}, that ensures the absolute speed V_a of the haulm particles, leaving the end of the blade, close to 60 m·s^{-1}. In this case the minimum value of the friction coefficient f, equal to 0.3, also increases the absolute speed.

Thus constructive and kinematic parameters have been found that provide the necessary conditions for efficient unloading of the cut sugar beet haulm by means of a blade spinner.

Conclusions

1. Differential equations of the movement of a haulm particle have been obtained that actually describe the process of loading sugar beet haulm using the working tools of the haulm removing machine – the movement along the surface of the blade of the loading mechanism. The equation of motion of the particle tops takes into account the design and power parameters of the loading process and is presented in a closed form. The equation of the movement of the haulm particle takes into account the design and power parameters of the loading process and is presented in a closed form.

2. The values of the blade length l, equal to 0.45 m, and of the angular speed ω, equal to 85 rad·s^{-1}, have been found, at which such a value of the absolute speed V_a of the haulm particle, leaving the end of the blade, equal to 60 m·s^{-1}, is reached, which will ensure the necessary transportation capabilities of the blade spinner when unloading the sugar beet haulm.

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